

Adaptive control in a resource management model

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Abstract

We cast a model of biological resource management as a problem of adaptive control in a nonlinear dynamical system. Optimisation of harvest, while ensuring that the resource population persists, is achieved through a simple algorithmic procedure which is remarkably robust under a variety of perturbations.

Keywords: Adaptive control; Optimization models; Resource management

1. Introduction

Reproduction models of biological populations have been widely used in the management of exhaustible resources (Clark, 1985, 1990). Typically, the models are used to evaluate various management programs that might be used with a view to the optimisation of some economic performance index (Clark, 1985, 1990). Recently, these bioeconomic models have received considerable attention as they form a possible basis for planning sustainable development (Hoppensteadt, 1982). The problems addressed by sustainable development basically involve deriving maximum profit while ensuring that the standing stock of the exhaustible resource does not vanish.

In this context Joshi and Gadgil (hereafter referred to as JG) studied a model of utilisation

of a single species of biological resource population by a community (Joshi and Gadgil, 1991). The aim of the community is to maximise the harvest of the resource population, without seriously depleting it. JG speculated that this could be achieved through a total protection of a fraction of the resource biomass from being harvested, in the form of uniformly distributed refugia. Through a combination of numerical and analytic studies they showed that such practices can indeed lead to the desired goal.

We recast this problem as one of *adaptive control*. Adaptive control, which is a method of self-regulation, whereby a system learns to conform to a desired paradigm, is an area of research in control theory (Santa Clara Workshop, 1987) that has seen considerable development in the past few decades. In particular, it is thought to occur commonly in biological and ecological contexts (Rosen, 1967, 1970; Toates, 1975). The basic aim of control theory is to make a dynamical

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system achieve a desired state. In nonlinear systems, where the dynamics can switch between different and competing attractors, the idea of adaptive control was suggested some years ago (Huberman and Lumer, 1990), and applied (Sinha et al., 1990) to model problems.

Ideas of control theory have also been applied to biological systems (Rosen, 1967,1970; Toates, 1975) to form simple models to explain their capacity to adapt to their changing environment (Holling, preprint). In the past few years, it has become increasingly common to model biological and ecological systems through nonlinear difference equations (see, e.g. May, 1976; Hoppensteadt, 1982) which are discrete in time. The dynamics described by the model equations is generally complex and can show a rich spectrum of dynamical behaviour as parameters in the model are varied (May, 1976; Feigenbaum, 1978,1979; Collet and Eckmann, 1980; Devaney, 1986): these can be simple attractors such as fixed points or limit cycles, as well as strange attractors with chaotic dynamics. However, it is an empirical observation that many real systems operate predominantly in a single behavioural mode, regardless of the intrinsic properties of the governing equations. Furthermore, the behaviour is maintained even when the parameters governing their dynamics can change due to environmental fluctuations. Consequently, studies on control of nonlinear systems (see Shinbrot et al., 1993 for a comprehensive review of various methods of control in chaotic systems) have centred on trying to ensure that the system reaches a desired state, which is known. This could be either a stable or an unstable cycle or a fixed point. In such situations a desired behaviour of the dynamical system is selected and control is then effected in order to ensure that the desired state is maintained.

In studies utilising adaptive control so far (Huberman and Lumer, 1990; Sinha et al., 1990), an attempt was made to reach a desired *known* state. In the present paper, we consider the problem of controlling a nonlinear system so that it reaches a desired state which is a priori not known, although the desired state is one which corresponds to a state variable reaching an extremum (the value of which is not known). Such

situations arise, for example, in harvesting problems where the aim is to optimise the harvest subject to certain constraints. We illustrate the control mechanism on the model studied by JG.

In this paper, we combine the ideas of refugia and adaptive control to suggest a practical scheme for resource management. It has been stressed most notably by Walters (1986) that resource management should be viewed as an adaptive process. There are several advantages of formulating the problem of sustainable development as one of adaptive control. Recently, much work has been done on the control of nonlinear systems, and several different methods of control have been developed (Shinbrot et al., 1993). This new perspective can yield fresh insight and suggest different tactics in resource management. Moreover, control can be applied not only to reach fixed points but also orbits of higher period and limit cycles. A wider field can therefore, be explored than the one limited to the harvesting of a single species. More realistic cases of multi-species problems (Yodzis, 1989), which may involve higher period orbits, become accessible. Even in the single species problem, there are certain populations of fishes (for example herrings and anchovies; Clark, 1985) which are subject to cycles of abundance and decline. Such situations can be dealt with by the present adaptive control mechanism. Since such an application is important in policy design (Walters, 1986) we suggest a specific methodology below.

In the following section, we briefly review the models of resource dynamics introduced by JG, both with and without refugia. In Section 3, the method of adaptive control is presented for the particular case at hand. Section 4 deals with the application of control to the resource dynamics. This is followed by a discussion of the robustness of the method in Section 5, and a summary in Section 6.

2. Resource dynamics

In this section we briefly review the model studied by JG, which describes the exploitation of

a resource population of a single species. Each time step consists of two phases: a growth phase governed by a logistic law

$$B'_t = rB_t(1 - B_t), \quad (1)$$

followed by a harvesting phase, where effort E_t results in a harvest H_t , obeying a law of diminishing returns

$$H_t = B'_t(1 - e^{-E_t}). \quad (2)$$

After the harvest the subsequent dynamics is given by

$$B_{t+1} = B'_t - H_t \quad (3)$$

(other growth laws such as modified logistic: $B'_t = rB_t(1 - B_t)^\gamma$; hyperbolic: $B'_t = rB_t(1 - (r - 1)B_t)$; or Ricker: $B'_t = B_t \exp(r(1 - B_t))$, all give similar results.)

When a fraction α of the resource biomass is exempted from being harvested in the form of refugia, the harvest-effort relation is modified as

$$H_t = (1 - \alpha)B'_t(1 - e^{-E_t}). \quad (4)$$

The resource population remaining after exploitation is

$$\begin{aligned} B_{t+1} &= (1 - \alpha)B'_t e^{-E_t} + \alpha B'_t \\ &= \alpha r B_t (1 - B_t) + (1 - \alpha) r B_t (1 - B_t) e^{-E_t} \\ &= r B_t (1 - B_t) [\alpha + (1 - \alpha) e^{-E_t}]. \end{aligned} \quad (5)$$

For constant effort, without refugia, the resource population reaches an equilibrium value given by

$$B_{eq} = 1 - \frac{e^E}{r}, \quad (6)$$

which implies that the resource biomass persists as long as the harvesting effort $E < E_{max} = \ln r$. In this model the equilibrium or sustainable value of the harvest is given as

$$H_{eq} = e^E - 1 - \frac{e^{2E}}{r} + \frac{e^E}{r}. \quad (7)$$

Clearly, H_{eq} vanishes for $E = 0$ and $E = E_{max}$ and attains a maximum value

$$\hat{H} = \frac{(r - 1)^2}{4r} \quad (8)$$

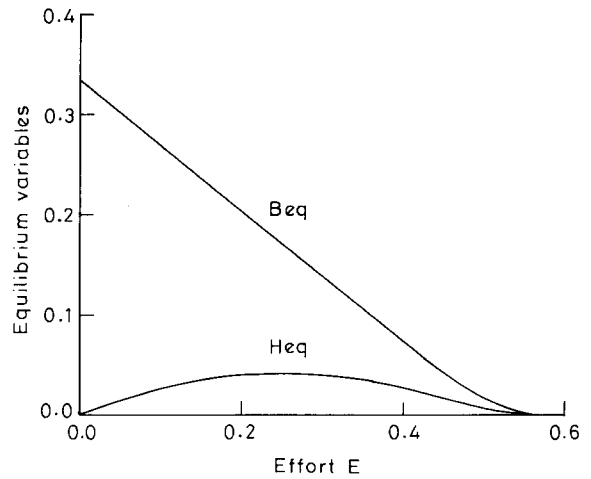


Fig. 1. Variation of the equilibrium biomass level, B_{eq} , and harvest, H_{eq} , as a function of effort, E , using Eqs. 6 and 7.

for the optimal effort $\hat{E} = \ln[(r + 1)/2]$ (see Fig. 1). When refugia are introduced the equilibrium biomass reaches the value

$$B_{eq} = 1 - \frac{1}{r[\alpha + (1 - \alpha)e^{-E}]}, \quad (9)$$

so that the resource population does not become extinct as long as α satisfies the inequality $r[\alpha - (1 - \alpha)e^{-E}] > 1$. For infinitely high harvesting effort, $B_{eq} \rightarrow (1 - 1/r\alpha)$, and the sustainable harvest reaches the value

$$H_{eq} \rightarrow \frac{(1 - \alpha)(r\alpha - 1)}{r\alpha^2}. \quad (10)$$

The refugium size $\hat{\alpha}$ that maximises the harvest is therefore

$$\hat{\alpha} = \frac{2}{1 + r} \quad (11)$$

and the maximum harvest thus obtained is $\hat{H} = (r - 1)^2/4r$. Note that this is the same value attained as in the case without refugia (this conclusion depends on the assumption that refugia are uniformly distributed over the resource population).

3. Adaptive control

It is well known (May, 1976; Feigenbaum, 1978,1979; Collet and Eckmann, 1980) that the

logistic growth law (Eq. 1) is capable of extremely rich dynamical behaviour, for $0 < B < 1$ and $0 < r \leq 4$. For $0 < r < 1$, the map is uninteresting as the dynamics is asymptotically attracted to the fixed point $B = 0$. At $r = 1$ there is a transcritical bifurcation, i.e. the two existing fixed points ($B = 0$ and $B_\mu = 1 - 1/r$) exchange their stability. Beyond $r = 3$ there is a series of period-doubling bifurcations accumulating at $r \approx 3.57$. At each stage of the bifurcation a 2^n period becomes unstable and 2^{n+1} period becomes stable. Beyond the accumulation point, orbits of other periods appear, with many completely aperiodic but bound orbits between them, until the birth of a 3-period orbit through a tangent bifurcation at $r = 3.8284 \dots$. After this the dynamics evolves on aperiodic orbits and for $r = 4$ we have fully developed chaos (Devaney, 1986).

By the control of such a system we mean the following. Say that the system operating at some parameter value $r = r_s$, is subjected to a sudden perturbation and the parameter value jumps to some $r = r_p \neq r_s$. This alters the behaviour of the system. If $1 < r_s < 3$ to begin with, the stable behaviour would be a period 1 orbit; if $r_p > 3$ after perturbation, the system would be locked into some higher period orbit. We would like to restore the system to its original dynamical behaviour. A difficulty is that for many a system the dynamics may have more than one possible stable orbit for the same parameter value. For example (Sinha et al., 1990), a system can operate at a fixed point or a limit cycle for the same set of parameter values. (This is common in dynamical systems of higher dimensions.) In order to restore the system to its original or desired behaviour a detailed knowledge of the dynamics is important.

A possible mechanism for robust self-regulation was suggested (Huberman and Lumer, 1990) through a simple and effective feedback algorithm (for one-dimensional systems) using an error signal proportional to the difference between the goal output and the actual output (Huberman and Lumer, 1990; Sinha et al., 1990; Sinha, 1991). This error signal tunes the parameters of the system, which readjust so as to reduce the error to zero. This algorithm was later generalised to higher dimensions and for systems with

more than one variable parameter (Sinha et al., 1990). Thus, for a general N -dimensional dynamical system

$$\dot{\vec{B}} \equiv \frac{d\vec{B}}{dt} = F(\vec{B}; \vec{r}; t), \quad (12)$$

where $\vec{B} \equiv (B_1, B_2, \dots, B_N)$ are variables and $\vec{r} \equiv (r_1, r_2, \dots, r_M)$ are parameters governing the dynamics, the prescription for adaptive control is

$$\dot{\vec{r}} = sg(\vec{B} - \vec{B}_s) \quad (13)$$

where \vec{B}_s is the desired steady state, s is the stiffness of control and $g(x)$ is an arbitrary function with the requirement that $g(0) = 0$. A variety of different forms of g have been tried and found to be equally effective (Sinha et al., 1990). This simple algorithm is remarkably robust even in the presence of external random noise. The time required for a system to readjust to its original behaviour, after a sudden perturbation, is inversely proportional to stiffness of control (Huberman and Lumer, 1990; Sinha et al., 1990; Sinha, 1991). This linearity of the recovery time is independent of the functional form of g and the external noise, and appears to be a universal feature of such control (Sinha et al., 1990; Sinha, 1991).

Orbits of period $n > 1$ can be controlled by generalising Eq. 13 to

$$\dot{\vec{r}} = s \prod_{i=1}^n g(\vec{B} - \vec{B}_i) \quad (14)$$

where \vec{B}_i 's are the elements of the stable period n orbit. If the function g is viewed as a derivative of some fictitious potential V and control is achieved through $g = 0$, the problem can be redefined as one of finding the extrema of V . With this viewpoint control is achieved when $V' = 0$. Orbits of period 2 could be controlled (Sinha et al., 1990) with the prescription of Eq. 14 and $g(B) = B$, but this form of $g(B)$ is not very stable for higher periods. This is so because with $g(B) = B$, the condition $g(0) = 0$ is a zero of order 1, i.e. only the first derivative of V vanishes. To increase the stability of the procedure, we require that several derivatives vanish as well; this can be achieved by taking g of the form

$$g(B) = B^p \quad (15)$$

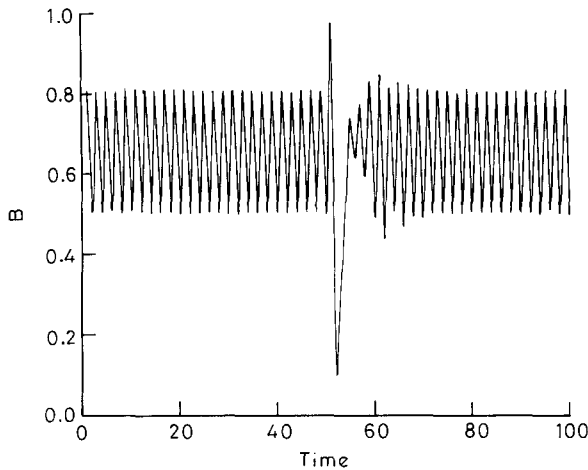


Fig. 2. Results of applying adaptive control (Eqs. 14, 15) on period 2, $\mu_s = 3.23604$, stiffness $s = 1.0$, exponent $p = 0.27$. The perturbed value of the control parameter was $\mu_p = 3.9$.

where p is some sufficiently large exponent. Using this form of g we were able to control higher period orbits for the logistic growth law (Eq. 1). Fig. 2 shows the control of a period 2 orbit. This control was found to be equally effective for other k -periods, $k = 3, 4, 5$ and 8 . In all these cases, the dynamics was first evolved with the

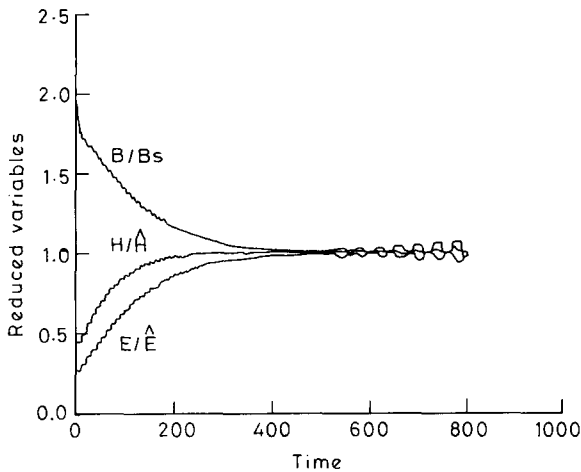


Fig. 3. Temporal variation of the reduced variables: harvest, \hat{H} , biomass level, B/B_s and effort, \hat{E} , when adaptive control algorithm (Eq. 18) is applied. \hat{H} is the maximum sustainable harvest (MSH), \hat{E} the effort corresponding to MSH and B_s is the asymptotic value of biomass level.

value of r appropriate to the existence of a stable period k orbit. At time $t = 50$, the control parameter was perturbed to a new value which was chosen to correspond to chaotic motion. Simultaneously, the control Eq. 14 was applied, and the system eventually recovered the original behaviour. As clear from the figure, the control is extremely effective since the recovery time is quite small. In fact, as mentioned above this time can be made smaller by increasing the stiffness of control. The drawback of too high a stiffness ($s \geq 100$) is that the dynamics may get arrested at an orbit which is close to the original behaviour, without actually reaching it exactly. Also, at such high stiffness constants it becomes more and more difficult to control a given behaviour as the dynamics becomes extremely sensitive to change in parameters. Thus the optimal control stiffness is of a moderately large magnitude ($s \geq 1-10$).

4. Variable harvesting strategy

As stated above, the problem of sustainable development aims at maximising profit while ensuring that the exhaustible resource persists. The model of resource dynamics studied by JG is such a problem, and the strategy they proposed for reaching the maximum sustainable harvest relied

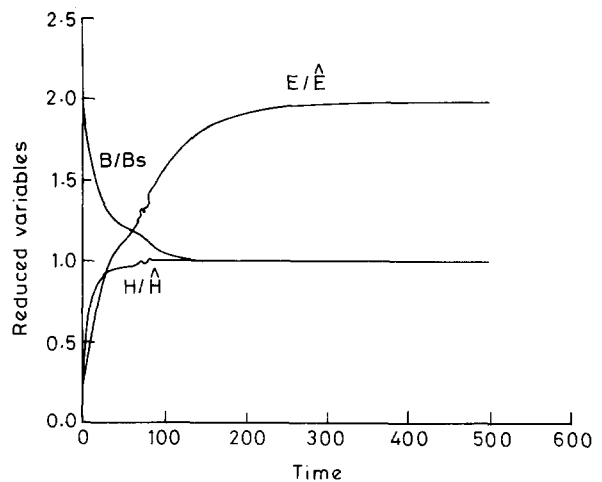


Fig. 4. As in Fig. 3, for the case when refugia (Eqs. 18, 19) are incorporated in the dynamics.

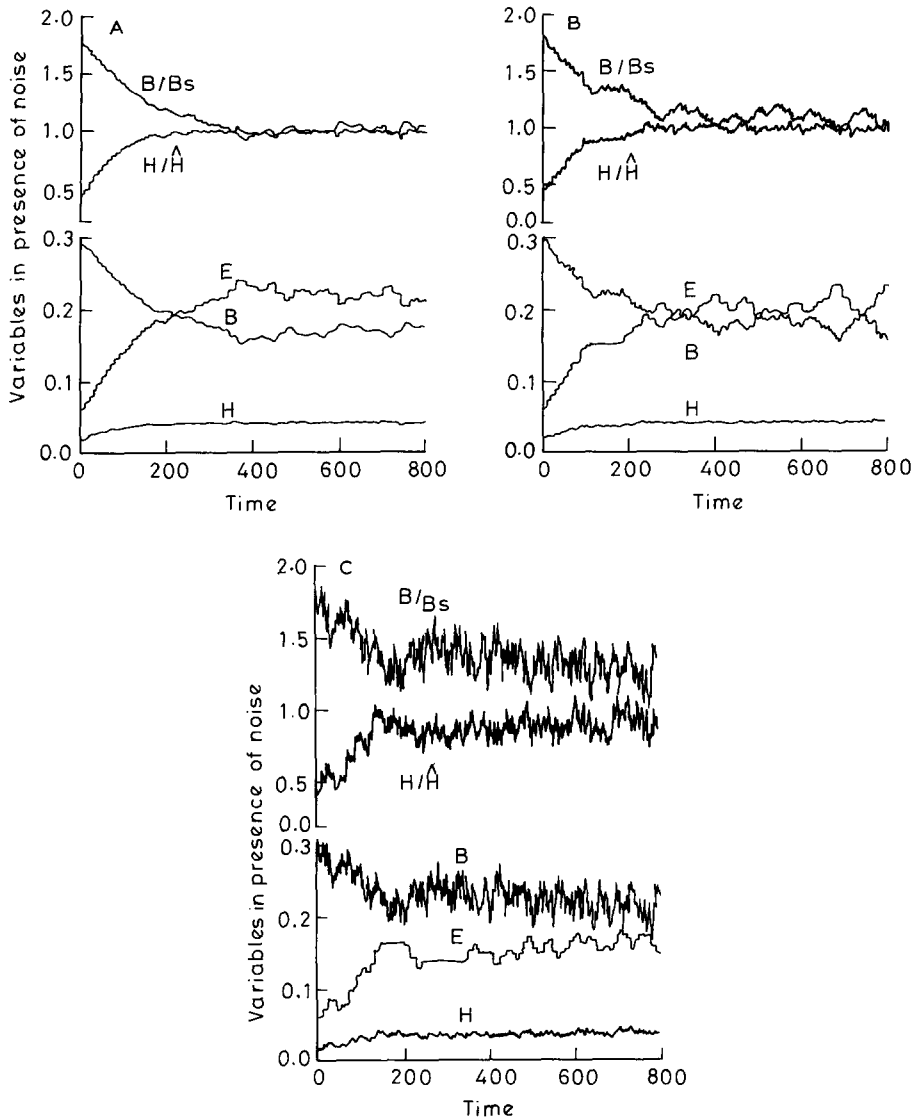


Fig. 5. Temporal variation of harvest, H , biomass level, B , and effort, E , for the case when random noise, of strength σ , is added in the dynamics (Eq. 21): (a) $\sigma/B_s = 0.006$; (b) $\sigma/B_s = 0.03$; (c) $\sigma/B_s = 0.3$. B_s is the asymptotic value of the resource biomass. For comparison reduced variables (as defined in Fig. 3) are also plotted.

on a simple feedback mechanism: start with a small effort E_0 , and vary it subsequently as follows

$$E_{t+n} = E_t + D_t$$

$$D_t \begin{cases} = xD_{t-n} & \text{if } H_t > H_{t-n} \\ = -xD_{t-n} & \text{if } H_t < H_{t-n} \end{cases} \quad (16)$$

where n is a time-lag, and x is a small positive factor. They find they were able to reach the desired goal of maximal harvest for $n \geq 10$, and $x = 0.01$.

To view this as a problem of adaptive control, note that the goal output is *unknown*. However, we know from Eqs. 6 and 7 that $dH_{eq}/dE = 0$ for $E = \hat{E}$, and as shown above, adaptive control can

be used in problems of maximisation. We thus propose the following variable harvesting strategy. Vary the effort as follows

$$\dot{E} = \epsilon \frac{\partial H}{\partial E} \quad (17)$$

where ϵ is stiffness of control. The derivative is taken at discrete intervals of time, as after every change of effort the system should be allowed to adjust to the new level of exploitation. In this case control is to be applied to reach a fixed point, and Eq. 17 is the appropriate substitute for Eq. 13, with $g = (\partial H/\partial E)$. This strategy indeed achieves the objective.

To drive the system faster towards the extremum, an exponential damping term can be added analogous to dynamical quenching (Stilinger and Weber, 1982)

$$\dot{E} = \epsilon \frac{\partial H}{\partial E} \exp\left(-\left|\frac{\partial H}{\partial E}\right|\right). \quad (18)$$

The effect of the exponential damping term is to reduce the fluctuations in the resource biomass level and the effort, once the harvest is sustained. A discretised version of Eq. 18 was used, i.e.:

$$E_{t+1} = E_t + \epsilon \frac{\partial H}{\partial E} \exp\left(-\left|\frac{\partial H}{\partial E}\right|\right). \quad (19)$$

These results are shown in Fig. 3, where it can be seen that with the dynamical quenching, fluctua-

tions in the resource population level have been significantly reduced from an amplitude of 0.165 (in the JG case) to 0.055 (Fig. 3). This is important as it ensures that the standing stock of the exhaustible resource is maintained at a more or less steady value, instead of undergoing large fluctuations which could lead to a temporary extermination of the stock. (This strategy works for all the different growth models described above. The basic requirement for this strategy to work is that the dynamics should involve a law of diminishing returns.)

We now consider the case with refugia. As stated above, this involves the total protection of a fraction α of the resource population. We have therefore an additional parameter to vary in order to reach our objective. Since a higher level of effort increases the risk of resource extermination, the procedure should involve an increase in the area under a refugium or a set of refugia, whenever the harvesting effort is stepped up, i.e. the change in α will be proportional to the change in effort. The decision rule (Eq. 19) is then modified by adding to a change in harvesting effort a change in refugium size as follows

$$\alpha_{t+1} = \alpha_t + s(E_t - E_{t-1}). \quad (20)$$

where s is an additional stiffness parameter. Again, our harvesting strategy drives the system to the maximum harvest with however a significantly reduced waiting time (see Fig. 4). With

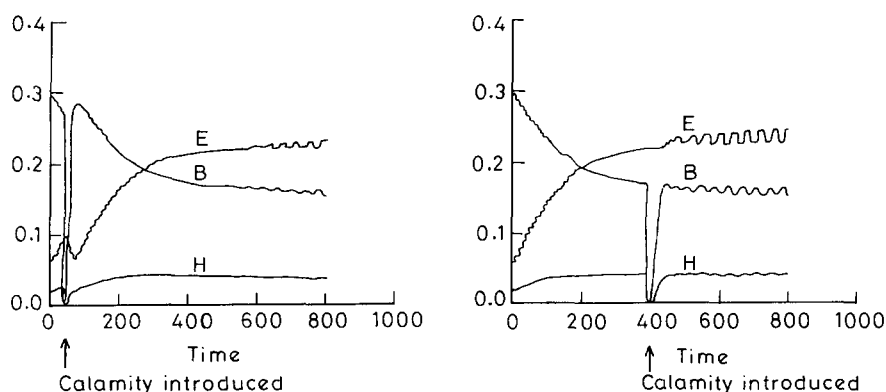


Fig. 6. Temporal variation of H , B and E , for the case when calamities are taken into account in the dynamics: calamity introduced in (a) the learning phase, (b) the stable phase. The variables represent the quantities defined above.

refugia, the fluctuations in the resource population are still further reduced.

5. Resilience of the control

The present control technique employed is robust to a variety of external perturbations such as external random fluctuations or calamities. Any realistic system will be subject to stochastic fluctuations in the environment, and therefore any proposed strategy should take into account this fact and be resilient to such fluctuations. One way in which random fluctuations can be introduced is by adding uniformly distributed random noise in the dynamics involving the growth of the resource biomass

$$B'_t = rB_t(1 - B_t) + \sigma\eta_t \quad (21)$$

where η_t is δ -correlated noise of strength σ . The strategy embodied in Eq. 18 is extremely effective in maintaining the maximum harvest as long as $\sigma/B_s \leq 0.1$, where B_s is the level of the standing stock when the maximum sustainable harvest is reached. Results are shown in Fig. 5, where the variables plotted are the harvest H , resource biomass B and effort E against time for different noise strengths σ . Fig. 5a shows the plot for low noise, $\sigma/B_s = 0.006$, Fig. 5b for moderate noise, $\sigma/B_s = 0.03$ and Fig. 5c for large noise amplitude, $\sigma/B_s = 0.3$. As can be seen clearly from the plots, fluctuations in the maximum harvest are small (Fig. 5c) even when the fluctuation in the resource biomass level is large.

The present algorithm is also resilient to calamities, i.e. instantaneous extermination of virtually the entire resource population. In a real situation, this could, for example, correspond to a year of drought or an attack by pests. In our model we introduce calamities by setting $B_t = 0.001$ at a time $t = t_c$. Fig. 6a illustrates the case in which the calamity was introduced in the learning phase, when the system has not yet reached the desired steady state. Fig. 6b shows the case when the calamity was introduced after the system has attained the maximum harvest and is operational for sufficient time. The introduction of calamities even at this phase is rapidly nullified

and the harvest rapidly attains its maximum operational value.

6. Discussion

The method suggested above can find practical application. As an example of a realistic case (of significant commercial interest), consider the Antarctic baleen whales, which were reduced to a fraction of their former abundance in the 1960s, due to overfishing by the whaling industries (Clark, 1985). From estimates of International Whaling Commission, the following values derive for the environmental carrying capacity, K , and the catchability coefficient, q ,

$$K = 400\,000 \text{ blue whales units (BWUs)}$$

$$q = 1.3 \times 10^{-5} \text{ per catcher day}$$

Let X_0 be the initial population of the whales in BWUs. (We work in dimensionless units, by rescaling the variables as $B = X/K$). The initial value of B is set to $B_0 = 0.2$, i.e. we start with a population which is one-fifth of the environmental carrying capacity. Indeed, this was the level to which the whaling industries had reduced the population of these mammals. In Eq. 1, r is taken to be 1.5, which roughly corresponds to an intrinsic growth rate of 4% per annum. The results of applying the dynamical equations in Eqs. 1–3 above give the following numbers for the first few iterations. If one starts with an initial effort of $e_0 = 2000$ catcher days per season, which in our dimensionless units becomes $E_0 = qe_0 = 0.026$, at time $t = 0$ we find $B'_0 = 0.24$ and $H_0 = 6.16 \times 10^{-3}$. At time $t = 1$, $B_1 = 0.234$ and changing the effort to $E_1 = 0.039$ (i.e. $e_1 = 3000$ catcher days per season), the harvest increases to $H_1 = 0.0103$. We then use Eq. 18 to calculate the new level of effort, which turns out to be $E_2 = 0.062$. This effort is held constant for the next ten seasons. At the end of this period the yield reaches a value of 0.0258 and the biomass level is 0.283 (i.e. 113 200 BWUs). This procedure is followed until equilibrium is reached, when the harvest level is seen to fluctuate about 0.0416 and the biomass level varies about an average 0.166 (i.e. 66 400

BWUs). The effort remains at a level 0.224 (i.e. about 17 230 catcher days per season) on average.

In order to implement the model including refugia, we start with a refugium of size zero and vary it subsequently as prescribed by Eq. 20. The other parameters remain the same as above except that now we need not wait for ten seasons before changing the effort. At equilibrium the harvest level is the same as above, but the fluctuations have been significantly reduced, so that it is constant, for all practical purpose. The biomass level is also approximately at the same level 0.158 (i.e. 63 200 BWUs) and is constant. On the other hand, effort level has gone up to 0.393 (i.e. around 30 230 catcher days per seasons). Thus, the example goes to show that judicious planning of resource exploitation using the methods of adaptive control and incorporation of refugia can increase the level of harvesting without necessarily endangering the survival of the species. Moreover, the equilibrium state is reached faster as the waiting time before every change of effort, has been reduced from ten seasons to one.

The above example shows how the control algorithm developed in this paper works in a practical problem. The robustness of this method, as exemplified by the studies with various perturbations, is an added advantage. Moreover, the recasting of the JG model as a control problem makes it more amenable to further exploration. One direction where this may prove useful is in the study of multi-species management, and multiple harvesting seasons. In these situations, the steady state is usually more complex than a simple fixed point. For example, in the case of multiple harvests more than one state variable is required to reach the maximum and it is a problem of interest whether an algorithm can be formulated so as to reach all the maxima. In the case of multi-species management, various kinds of steady states can exist, such as limit cycles (for two species with Lotka–Volterra interactions, for example). As pointed out with the example of the logistic map, the control mechanism presented in this paper is capable of controlling orbits of arbitrary period; it may be useful to explore the possibility of applying this control in the harvesting of several species and deducing efficient

adaptive mechanisms in policy design for complex problems.

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References

- Clark, C.W., 1985. *Bioeconomic Modelling and Fisheries Management*. Wiley, New York.
- Clark, C.W., 1990. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*, 2nd ed. Wiley, New York.
- Collet, P. and Eckmann, J.-P., 1980. *Iterated Maps on the Interval as Dynamical Systems*. Progress in Physics, Vol. 1. Birkhäuser Verlag, Basel.
- Devaney, R.L., 1986. *Introduction to Chaotic Dynamical Systems*. Wiley, New York.
- Feigenbaum, M.J., 1978. Quantitative universality for a class of nonlinear transformations. *J. Stat. Phys.*, 19: 25–52.
- Feigenbaum, M.J., 1979. The universal metric properties of nonlinear transformations. *J. Stat. Phys.*, 21: 669–706.
- Holling, C.S. *Sustainability: The Cross-Scale Dimension* (pre-print).
- Hoppensteadt, F.C., 1982. *Mathematical Methods of Population Biology*. Cambridge University Press, London.
- Huberman, B.A. and Lumer, H.L., 1990. Dynamics of adaptive controls. *IEEE Trans. Circuits Syst.*, 37: 547–550.
- Joshi, N.V. and Gadgil, M., 1991. On the role of refugia in promoting prudent use of biological resources. *Theor. Popul. Biol.*, 40: 211–229.
- May, R.M., 1976. Simple mathematical models with very complicated dynamics. *Nature*, 261: 459–467 (and references therein).
- Rosen, R., 1967. *Optimality Principles in Biology*. Butterworth, London.
- Rosen, R., 1970. *Dynamical Systems Theory in Biology*, Vol. 1. Wiley-Interscience, New York.
- Santa Clara Workshop, 1987. Challenges to control: a collective view. *IEEE Trans. Autom. Control*, 32: 275.
- Shinbrot, T., Grebogi, C., Ott, E. and Yorke, J.A., 1993. Using small perturbations to control chaos. *Nature*, 363: 411–417 (and references therein).
- Sinha, S., 1991. An efficient control algorithm for nonlinear systems. *Phys. Lett. A*, 156: 475–478.
- Sinha, S., Ramaswamy, R. and Subba Rao, J., 1990. Adaptive control in nonlinear dynamics. *Physica D*, 43: 118–128.

Stillinger, F.H. and Weber, T.A., 1982. Hidden structure in liquids. *Phys. Rev. A*, 25: 978–989.

Toates, F., 1975. *Control Theory in Biology and Experimental Psychology*. Hutchinson, London.

Walters, C., 1986. *Adaptive Management of Renewable Resources*. Macmillan Publishing Company, New York.

Yodzis, P., 1989. *Introduction to Theoretical Ecology*. Harper and Row Publishers, New York.